

1. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} f \frac{\sin 2x}{x} \quad 3f' 2 \quad 9$

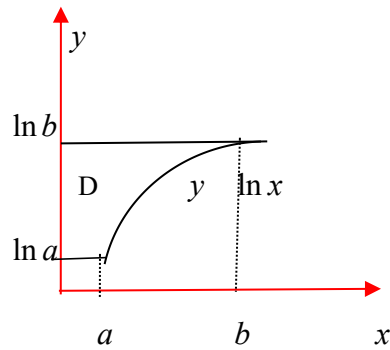
2. $K \frac{|y''|}{1 - y'^{2/3}} = \frac{2a}{1 - y'^{2/3}} \quad |y'| \quad y' = 2ax - b = 0 \quad x = \frac{b}{2a}$

3. $\tan y = x \quad y, \quad \sec^2 y dy = dx \quad dy = \cot^2 y dx$

4. $\cos \langle a, b \rangle = \frac{a \cdot b}{|a||b|} = \frac{3 \cdot 2 \cdot 2}{\sqrt{9} \cdot 1 \cdot 4\sqrt{1} \cdot 1 \cdot 4} = \frac{3}{2\sqrt{21}}$

1. $\lim_{x \rightarrow 0} 1 - \cos x \frac{3}{\cos x} = 2^3 = 8$

2. $A = \int_{\ln a}^{\ln b} e^y dy = \int_0^a \ln b - \ln a dx = \int_a^b \ln b - \ln x dx$



3. $M = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{1 - x^2} \cos^4 x dx = 0$

$N = 2 \int_0^{\frac{\pi}{2}} \cos^4 x dx = 0, P = 2 \int_0^{\frac{\pi}{2}} \cos^4 x dx = 0$
 $M = N = P$

4. $x = x_0 + U, \quad U = x_0 + \delta, \quad f(x) = f(x_0) + \frac{f'''(x_0)}{3!} (x - x_0)^3 + \dots$
 $f'(x) = f'''(x_0) (x - x_0)^2, \quad f''(x) = 2f'''(x_0) (x - x_0), \quad f'''(x) = f'''(x_0)$

1. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-2h)}{h} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} - 2 \lim_{h \rightarrow 0} \frac{f(a) - f(a-h)}{2h} = f'(a) - 2f'(a) = -f'(a)$

2. $\lim_{x \rightarrow \frac{\pi}{2}} x \frac{\pi}{2} \cot 2x = \lim_{u \rightarrow 0} \frac{u}{\tan 2u} = \frac{1}{2}$

$$1. \int \frac{10^{2\arccos x}}{\sqrt{1-x^2}} dx = \frac{1}{2} \int 10^{2\arccos x} d(2\arccos x) = \frac{10^{2\arccos x}}{2 \ln 10} + C$$

$$2. \int_0^2 f(x) dx = \int_1^2 f(u) du = \int_1^2 \frac{1}{e^x} dx = \ln 2 - \int_1^2 \frac{1}{x} dx = \ln 2 - \left[\frac{e^x}{1} \right]_1^2 = \ln 2 - \frac{e^2 - e}{1}$$

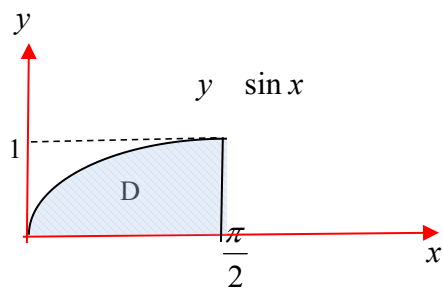
$$3. \int_0^{\frac{\pi}{2}} e^{2x} \sin 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \sin 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \cos 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \cos 2x dx = \frac{1}{2} e^{\frac{\pi}{2}} \sin \frac{\pi}{2} = \frac{1}{2} e^{\frac{\pi}{2}}$$

$$1. x_t \Big|_{t=0} = 2e^t, y_t \Big|_{t=0} = e^t, x_0 = 2, y_0 = 1, K = \frac{2}{1} = 2, K = \frac{1}{2}$$

$$y = 1 - 2x^2, y = 2x - 5, y = 1 - \frac{1}{2}x^2, y = \frac{1}{2}x$$

$$2. V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h^2 \cdot h^2 = V = \frac{400}{3} \pi = \pi h^2 \Rightarrow h = \frac{20}{\sqrt{3}}$$

$$3. V = 2\pi \int_0^{\frac{\pi}{2}} xy dx = 2\pi \int_0^{\frac{\pi}{2}} x \sin x dx = 2\pi \left[-x \cos x + \sin x \right]_0^{\frac{\pi}{2}} = 2\pi$$



$$x_n = \sqrt{2 - x_{n-1}}, \lim_{n \rightarrow \infty} x_n = A, A = \sqrt{2 - A} \Rightarrow A^2 = 2 - A \Rightarrow A^2 + A - 2 = 0$$

$$\left| \frac{x_{n-1} - 2}{\sqrt{2 - x_{n-1}}} \right| = \frac{1}{4} |x_{n-1} - 2| = \frac{1}{4^2} |x_{n-2} - 2| \dots = \frac{1}{4^{n-1}} |x_1 - 2| \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$F'(x) = f(x) \cdot \frac{1}{f(x)} = 2 \cdot 0 = F(x)$$

$$2 \int_b^a f(t) dt = 0, \int_a^b \frac{1}{f(t)} dt = 0$$

$$F(b) - F(a) = 0, \quad F(x) = \dots$$

$$e^x f(x) - f'(x) = 0 \Rightarrow e^x f(x)' = 0 \Rightarrow e^x f(x) = C$$

$$f(0) = 1 \Rightarrow C = 1 \Rightarrow f(x) = e^{-x}$$

08-09 A1

$$1. \lim_{x \rightarrow 0} x \sin \frac{1}{x} = \frac{1}{x} \sin x = 0 \cdot 1 = 1$$

$$2. y' = f'(\ln x) \cdot \frac{1}{x} \Rightarrow dy = \frac{f'(\ln x)}{x} dx$$

$$3. \int e^x \cdot 3 \cos x dx = e^x \cdot 3 \sin x + C$$

$$4. \int_a^a x^3 \sin^3 x dx = 0$$

$$1. \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = f'(0) = 2$$

$$2. y = x \Rightarrow y' = 1$$

$$3. \overrightarrow{NM} = (3, 4, 5), \overrightarrow{NP} = (1, 2, 2) \Rightarrow \cos \angle MNP = \frac{\overrightarrow{NM} \cdot \overrightarrow{NP}}{|\overrightarrow{NM}| |\overrightarrow{NP}|} = \frac{3 \cdot 1 + 8 \cdot 2 + 10 \cdot 2}{\sqrt{50} \cdot 9} = \frac{\sqrt{2}}{2} \Rightarrow \angle MNP = \frac{\pi}{4}$$

$$4. f'''(x) = 0, f''(0) = 0 \Rightarrow f''(x) = 0 \Rightarrow f'(x) = \frac{f(1) - f(0)}{1 - 0} = f'(1) = \xi = f'(1) = \xi = 0,1$$

$$5. \int_0^1 f(x) dx = A \Rightarrow \int_0^1 f(x) dx = 2A = 0,1 \Rightarrow A = \frac{1}{2} \Rightarrow 2A = A = \frac{1}{2}$$

$$1. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{\cos x \sin^3 x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^3} = \frac{1}{2}$$

2.

$$3. \lim_{x \rightarrow 0} \sin x^x = \lim_{x \rightarrow 0} e^{x \ln \sin x} = \lim_{x \rightarrow 0} e^{\frac{\ln \sin x}{\frac{1}{x}}} = \lim_{x \rightarrow 0} e^{\frac{x^2}{\tan x}} = e^0 = 1$$

$$1. x' = 2t, y' = \sin t. \frac{dy}{dx} = \frac{y'}{x'} = \frac{\sin t}{2t} \cdot \frac{d^2 y}{dx^2} = \frac{d}{dt} \frac{dy}{dx} \frac{dt}{dx} = \frac{\cos t \cdot 2t - 2 \sin t}{4t^2 \cdot 2t} = \frac{\sin t - t \cos t}{4t^3}$$

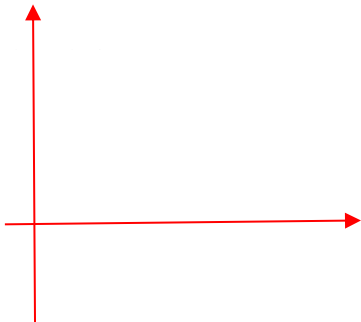
$$2. \int \frac{x e^{x^2}}{1 - 2e^{x^2}} dx = \frac{1}{2} \int \frac{d e^{x^2}}{1 - 2e^{x^2}} = \frac{1}{4} \ln |1 - 2e^{x^2}| + C$$

$$3. \int_1^4 \frac{\ln x}{\sqrt{x}} dx = 2\sqrt{x} \ln x \Big|_1^4 - \int_1^4 \frac{2}{\sqrt{x}} dx = 8 \ln 2 - 4\sqrt{x} \Big|_1^4 = 8 \ln 2 - 4$$

$$\begin{aligned} F(x) &= \sin x \tan x = 2x, F'(x) = \cos x \sec^2 x = 2 \cos x \tan^2 x + 1 \\ G(x) &= \cos x - 1 \tan^2 x, G'(x) = -\sin x - 2 \tan x \sec^2 x = -\sin x - 2 \sec^3 x \\ \therefore 0 &= \cos x - 1 \tan^2 x = 0, \frac{\pi}{2} \sec x = \frac{1}{\cos x} \Rightarrow \sec^3 x = 1 \Rightarrow 0 \\ G'(x) &= 0, G(0) = 0, G(x) = 0, f'(x) = 0, f(0) = 0, f(x) = 0 \\ \sin x \tan x &= 2x \end{aligned}$$

$$\begin{aligned} \int_0^x f(x) dx &= \int_0^x f(x) dx = \int_0^x f(x) dx = 1 - \int_0^x f(x) dx \\ x=0: \int_0^0 f(x) dx &= 1 - \int_0^0 f(x) dx = 1 - \int_0^0 \frac{1}{2} \sin x dx = 1 - \frac{1}{2} \cos x \Big|_0^0 = \frac{1}{2} - \frac{1}{2} \cos x \\ x=\pi: \int_0^\pi f(x) dx &= 1 - \int_0^\pi f(x) dx = 1 - \int_0^\pi \frac{1}{2} \sin x dx = \frac{\pi}{2} - \frac{1}{2} \cos x \Big|_0^\pi = \frac{\pi}{2} - \frac{1}{2} \cos \pi = \frac{\pi}{2} + \frac{1}{2} \end{aligned}$$

$$\begin{aligned} f(x) &= \int_a^x f(x) dx = f(a) + \int_a^a f(x) dx = 0 \\ x=a: f(a) &= C e^a = 0 \Rightarrow C = 0 \\ f(x) &= C e^x, x=a \end{aligned}$$



$$\begin{aligned}
 S &= \int_0^1 ax + bx^2 dx = \frac{a}{2} \frac{b}{3} \frac{4}{9} = b \frac{4}{3} \frac{3}{2} a \\
 V &= \pi \int_0^1 y^2 dx = \pi \int_0^1 a^2 x^2 dx = \frac{4}{3} \frac{3}{2} a^2 x^4 = 2a \frac{4}{3} \frac{3}{2} a x^3 dx = \pi \frac{1}{3} a^2 \frac{1}{5} \frac{4}{3} \frac{3}{2} a^2 = \frac{a}{2} \frac{4}{3} \frac{3}{2} a \\
 V' &= \pi \frac{2}{3} a \frac{3}{5} \frac{4}{3} \frac{3}{2} a = \frac{1}{2} \frac{4}{3} \frac{3}{2} a = \frac{3}{4} a = \pi \frac{2}{5} a \frac{2}{15} = 0 = a \frac{1}{3} b \frac{4}{3} \frac{3}{2} \frac{1}{3} \frac{5}{6}
 \end{aligned}$$

09-10 A1

$$1. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$$

$$2. \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0 \quad f(0) = 0,$$

$$\lim_{x \rightarrow 0} f'(x) = 0 \quad \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{x} = 0 \quad \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0 \quad f'(0) = 0$$

$$3. \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} x + 1 = 2$$

$$\lim_{x \rightarrow 1} f'(x) = 2x = 2,$$

4.C

$$1. \lim_{x \rightarrow 1} 1 - \frac{1}{x} = 0 \quad \lim_{x \rightarrow 1} 1 - \frac{1}{x} = 0 \quad e^3$$

$$2. \lim_{x \rightarrow 0} \frac{e^{3x \sin x} - 1}{\tan x^2} = \lim_{x \rightarrow 0} \frac{2x \sin x}{\tan x^2} = 2$$

$$3. k \lim_{x \rightarrow 3} \frac{y}{x} = \lim_{x \rightarrow 3} \frac{x}{3x - 1} = \frac{1}{3} \quad b \lim_{x \rightarrow 3} y = kx = \lim_{x \rightarrow 3} \frac{3x}{9x - 3} = \frac{1}{3}$$

$$y = \frac{1}{3} x + 1$$

$$4. y' = 2 \frac{\ln 1 - x}{1 - x}, \quad dy = 2 \frac{\ln 1 - x}{x - 1} dx$$

$$5. y' = x e^{\frac{x^2}{2}} = 0 \quad x = 0 \quad y'' = e^{\frac{x^2}{2}} - x^2 e^{\frac{x^2}{2}} = 0 \quad x^2 = 1 \quad 0 \quad x = 1$$

$$6. x f''(x) dx = x f'(x) = f'(x) dx = x f'(x) = f(x) + C$$

$$7. \int_0^5 e^{5x} dx = \frac{1}{5} e^{5x} \Big|_0^5 = \frac{1}{5}$$

$$8. \bar{y} = \frac{1}{\pi} \int_0^{\pi} \sin x dx = \frac{2}{\pi}$$

$$9. \int_1^2 |x| \sin x x^2 dx = 2 \int_0^1 x^3 dx = \frac{1}{2}$$

$$10. \int_0^1 f(x) dx = A, f(x) = x - 2A \quad 0,1 \quad A = \frac{1}{2} \quad 2A = A = \frac{1}{2}$$

$$1. x' = 2t - 1, y' = \cos t \quad \frac{dy}{dx} = \frac{y'}{x'} = \frac{\cos t}{2t - 1}, \frac{d^2 y}{dx^2} = \frac{d}{dt} \frac{dy}{dx} \frac{dt}{dx} = \frac{\sin t \cdot 2t - 1 \cdot 2 \cos t}{(2t - 1)^3}$$

$$2. \sin xy = \ln y = x \quad \frac{dy}{dx} = \frac{dx}{dy} \quad dx, 0,1$$

$$\frac{dy}{dx} = \frac{y}{x} \quad y = x + 1$$

$$1. \lim_{x \rightarrow 0} \frac{e^x - 1 - 2x^{\frac{1}{2}}}{\ln 1 - x^2} = \lim_{x \rightarrow 0} \frac{e^x - 1 - e^{\frac{1}{2} \ln 1 - 2x}}{x^2} = \lim_{x \rightarrow 0} \frac{e^x - 1 - e^{\frac{1}{2} \ln 1 - 2x} - x}{x^2} = \lim_{x \rightarrow 0} \frac{\frac{1}{2} \ln 1 - 2x - x}{x^2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{1 - 2x}{2x} = \frac{1}{4}$$

$$2. n \frac{n}{n^2} = A = n \frac{1}{1 - n^2} \quad A = 1$$

$$x \quad x \quad \int_0^x f(x) dx = -x^2$$

$$x \quad x \quad \int_0^x f(x) dx = \int_0^3 f(x) dx = \int_3^x f(x) dx = \frac{3}{4} x^2 - \frac{x}{2} dx = 2x - 3 - \frac{1}{4} x^2$$

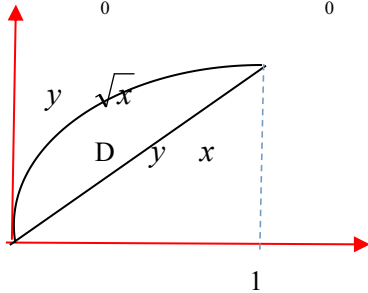
$$\frac{dy}{dx} \sin x = y \ln y \quad \frac{dy}{y \ln y} = \frac{dx}{\sin x} \quad \ln \ln y = \ln |\csc x - \cot x| + \ln c$$

$$\ln y = c \csc x - \cot x \quad y = \pi/2 e, \quad c = 1 \quad y = \csc x - \cot x$$

$$2. x^2 - 1 y' = 2xy \cos x, \quad x^2 - 1 y' = \cos x \quad x^2 - 1 y = \sin x + C$$

$$A = \int_0^1 \sqrt{x} \cdot x \, dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1 = \frac{1}{2} x^2 \Big|_0^1 = \frac{1}{6}$$

$$V = \pi \int_0^1 \sqrt{x} \cdot x^2 \, dx = \pi \int_0^1 x \cdot x^2 \cdot x^{\frac{3}{2}} \, dx = \frac{13\pi}{25}$$



$$2. F(x) = f(x) = \sin x \quad F(0) = f(0) = 0, \quad F(\pi/2) = f(\pi/2) = 1 = 0$$

$$\xi = 0, \frac{\pi}{2}$$

$$F'(x) = f'(x) = \cos x \neq 0 \quad f'(x) = \cos x$$

10-11 A1

$$1. \lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x} = \lim_{x \rightarrow 0} \frac{1-x - 1+x}{x \sqrt{1-x} \sqrt{1+x}} = \frac{2x}{2x} = 1$$

2.D

$$3. F(x) = e^x - x^2 \quad F'(x) = e^x - 2x \quad F(0) = 1 - 0 = 1, \quad F(1) = e - 2 = 0$$

$$4. f(x) \quad , \quad F(x) = G(x) + C$$

$$1. \lim_{x \rightarrow 1} e^{\frac{1}{x-1}} = e = 0$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x}{x-1} = \lim_{x \rightarrow 0} \frac{3x}{2x} = \frac{3}{2}$$

$$3. \quad \int e^y dy = y dx - x dy \quad 0 \quad dy = \frac{y}{e^y} dx$$

$$4. \lim_{x \rightarrow 0} \lim_{x \rightarrow 0} x^{\frac{1}{2}} \lim_{x \rightarrow 0} \frac{1}{x} = \lim_{x \rightarrow 0} \frac{\frac{2}{3} e^{\frac{1}{2}}}{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{2e^{\frac{1}{2}}}{x}$$

$$11. \quad dy \frac{\arctan x}{1-x^2} dx \quad y \Big|_{x=0}^1 = \frac{1}{2} \arctan x^2 + C \quad y \Big|_{x=0}^1 = 1, \quad C = 1 - 2y \arctan x^2 - 2$$

$$2. \quad \frac{dy}{dx} = \frac{1}{\sqrt{9-x^2}} \quad y = \arcsin \frac{x}{3} + C$$

0,1 $C = 1.$ $y = \arcsin \frac{x}{3} + 1$

$$2. \quad \frac{xy' - y}{x^2} = \cos x. \quad \frac{y'}{x} = \cos x \quad \frac{y}{x} = \sin x + C$$

$$1. \quad x=0. \quad x^2 = \int_0^x 2x \cdot \frac{3}{2} x^2 dx = x^2 \cdot \frac{1}{2} x^3 = \frac{1}{2}$$

$$0. \quad x=1. \quad x^2 = \int_0^1 2x \cdot \frac{3}{2} x^2 dx = \int_0^1 \frac{1}{e^x} dx = \frac{1}{2} \int_0^1 \frac{e^x - e^x}{1 - e^x} dx = \frac{1}{2} (x \ln 1 - e^x)$$

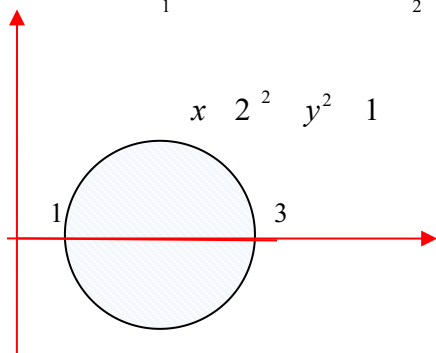
$x^2 = \frac{1}{2} x^3 = \frac{1}{2}, \quad 1 - x = 0$

$\frac{1}{2} x \ln 1 - e^x, \quad 0 - x = 1$

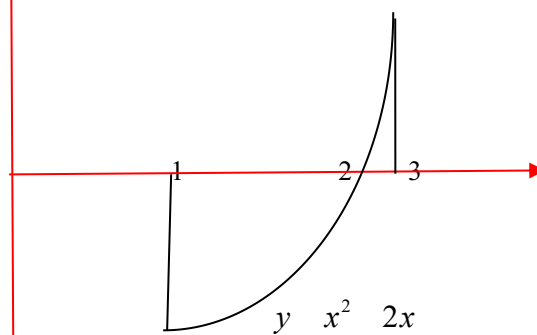
$$1. \quad V = 4\pi \int_1^3 x \sqrt{1-x^2} dx = 4\pi \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} 2 \sin t \cos^2 t dt = 4\pi \int_0^{\frac{\pi}{2}} 2 \cos^2 t dt = 2\pi^2$$

$$2. \quad A = \int_0^2 |y| dx = \int_0^2 2x - x^2 dx = \frac{4}{3}$$

$$2. \quad V = 2\pi \int_1^2 x \cdot 2x - x^2 dx = 2\pi \int_2^3 x x^2 - 2x dx = 9\pi$$



(1)



(2)

$$1. \quad \int_0^2 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx = \int_0^2 f(x) dx$$

$$\left| \int_0^2 f(x) dx \right| = \left| \int_0^1 f(x) dx + \int_1^2 f(x) dx \right| = \left| \int_0^1 f'(\xi_1) x dx + \int_1^2 f'(\xi_2) x dx \right|$$

$$\left| \int_0^1 f(\xi_1) x dx \right| \left| \int_1^2 f(\xi_2) 2x dx \right| = M \int_0^1 x dx + M \int_1^2 2x dx = \frac{1}{2}M + \frac{1}{2}M = M$$

$\xi_1 \in [0,1], \xi_2 \in [1,2]$

$$2 \int_0^1 f'(\xi_1) x^p dx = f(\xi_1) x^{p+1} \Big|_0^1 = f(\xi_1)$$

$$\int_1^2 f''(\xi) x^q dx = f'(\xi_2) x^{q+1} \Big|_1^2 = f'(\xi_2) (2^{q+1} - 1)$$

$\xi_1 \in [0,1], \xi_2 \in [1,2], \xi \in [1,2], a, b$

11-12 A1

1. --D

$$2. \lim_{x \rightarrow 0} \frac{f^2(x) - x^2}{x^2 - f^2(x)}$$

7.0

$$8. dA = \frac{1}{2} \rho^2 \theta d\theta \quad V = \int_c^d \pi \phi^2 y dy$$

$$1. \lim_x \frac{2x-1}{2x-1}^{2x} = \lim_x \left(1 + \frac{2}{2x-1}\right)^{2x} = \lim_x \left(1 + \frac{2}{2x-1}\right)^{\frac{2x-1}{2} \cdot \frac{4x}{2x-1}} = e^{\frac{1}{2}}$$

$$2. n \frac{n}{n^2} = \frac{n}{n\pi} \quad A = n \frac{n}{n^2} = \frac{n}{\pi}$$

$$, \quad A = 1$$

$$3. x' = 1 + \frac{1}{1-t} \cdot \frac{t}{1-t}, y' = 3t^2 - 2t \quad \frac{dy}{dx} = \frac{y'}{x'} = \frac{3t^2 - 2t - \frac{t}{1-t}}{1 + \frac{t}{1-t}} = \frac{3t^2 - 2t - t}{1-t} = \frac{3t^2 - 3t}{1-t} = 3t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \frac{dy}{dx} \cdot \frac{dt}{dx} = \frac{6t - 3}{1-t} \cdot \frac{1}{1-t} = \frac{6t - 3}{(1-t)^2}$$

$$F(x) = 2x \ln \frac{1+x}{1-x}$$

$$F'(x) = 2 \left(\frac{1}{1+x} - \frac{1}{1-x} \right) = 2 \frac{x^2 - 1}{1-x^2} = 0 \quad 0, 1$$

$$F(0) = 0, \quad F(0) = 0 \quad 2x \ln \frac{1+x}{1-x} = 0 \quad e^{2x} = \frac{1+x}{1-x}$$

$$1. \int \ln \sin x \csc^2 x dx = \int \ln \sin x \cot x - \cot^2 x dx = \int \ln \sin x \cot x dx - \int \csc^2 x dx$$

$$= \int \ln \sin x \cot x dx - \cot x - x + C$$

$$= \int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} e^{2x} + C = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

$$2. \int \frac{x^2}{x^2 - 2x + 3} dx = \int \frac{2x^2 - 6}{x^2 - 2x + 3} dx = \int \frac{d(x^2 - 2x + 3) - 2x + 3}{x^2 - 2x + 3} dx = \int \frac{1}{x^2 - 2x + 3} dx$$

$$= \frac{1}{2} \ln |x^2 - 2x + 3| - \frac{3}{\sqrt{2}} \arctan \frac{x-1}{2} + C$$

$$3. \int_0^1 f(x) dx = A \quad 0, 1, \quad A = \int_0^1 \frac{1}{x^2} dx = A = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$A = \frac{\pi}{4} - \frac{\pi}{2} = A = \frac{\pi}{4} - \frac{\pi}{2\pi}$$

$$1. \int_1^x 3f(t) dt = \int_1^x f(t) dt = \int_1^x 1 - t^2 dt = t - \frac{1}{3} t^3 \Big|_1^x = x - \frac{1}{3} x^3 - \frac{2}{3}$$

$$x^3, F(x) = \int_1^x f(t) dt = \int_1^3 f(t) dt + \int_3^x f(t) dt = \int_1^3 f(t) dt + t \cdot \frac{1}{3} t^3 \Big|_3^x = \frac{16}{3}$$

$$F(x) = x \cdot \frac{1}{3} x^3 - \frac{2}{3}, \lim_{x \rightarrow 3} F(x) = 3 \cdot \frac{1}{3} 3^3 - \frac{2}{3} = \frac{16}{3} = F(3)$$

$$f(3) = 0, f(3) = 8 = F(x) = x^3$$

$$7.4x^2 - y^2 = 4, 8x - 2yy' = 0 \implies y' = \frac{4x}{y} \implies Y = y = \frac{4x}{Y} \implies X = x$$

$$X = 0, Y = \frac{4}{y} \cdot Y = 0 \implies X = \frac{1}{x}$$

$$S = \frac{1}{2} XY = \frac{1}{4} 2\pi = \frac{2}{xy} = \frac{\pi}{2} = \frac{2}{x\sqrt{4-4x^2}} = \frac{\pi}{2} = \frac{1}{x\sqrt{1-x^2}} = \frac{\pi}{2}$$

$$f'(\xi) = \frac{f(b) - f(a)}{b - a} = \frac{f(c) - f(b)}{c - b}, \quad \xi \in (a, b)$$

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$$1. \lim_{x \rightarrow 0} \frac{1}{x} = \lim_{x \rightarrow 0} \frac{1}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \quad D$$

$$2. \lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} \frac{1 - e^{2x^2}}{1 - e^{x^2}} = 1 \quad y = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - e^{x^2}}{1 - e^{x^2}} = \frac{2}{0} \quad x = 0$$

3. D

$$4. f'(\sin^2 x) = 1 - \cos^2 x = f'(x) = 1 - x$$

$$f(x) = x - \frac{1}{2}x^2 + C$$

$$1. \lim_{x \rightarrow 1} \frac{x - 1}{x} = \lim_{x \rightarrow 1} \frac{1}{x} = \frac{1}{1} = 1 \quad e^2$$

$$2. y' = \frac{1}{x} \cdot y'' = \frac{1}{x^2} \cdot y''' = 1 - \frac{2}{x^3} \dots y^{(n)} = 1 - \frac{n-1!}{x^n}$$

$$3. \int x dy - y dx = e^{x-y} dx - dy$$

$$x dy - y dx = xy dx - dy = dy \frac{xy - y}{x - xy} dx$$

$$4. y' = e^x - x e^x \cdot y'' = e^x - x e^x = 2 - x e^x = 0 \quad x = 2$$

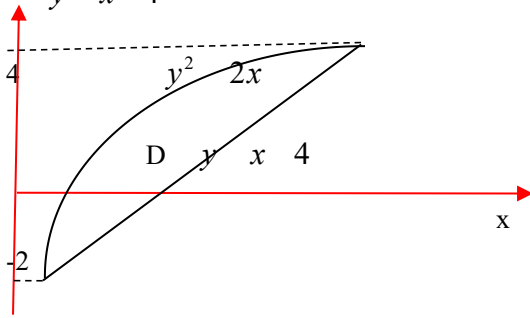
$$x = 0 \cdot y'' = 2, 2e^2$$

$$5. \frac{d}{dx} \int_0^{x^2} \ln 1 - t dt = 2x \ln 1 - x^2$$

$$6. \int_e \frac{dx}{x \ln^2 x} = \int_e \frac{d \ln x}{\ln^2 x} = \frac{1}{\ln x} \Big|_e = 1$$

$$7. \rho(x_0) = \lim_{x \rightarrow x_0} \frac{m(x) - m(x_0)}{x - x_0} = m'(x) \Big|_{x=x_0} = m'(x_0)$$

$$8. \int_{y=2}^{y=4} \int_{x=y^2}^{x=4} \frac{y^2}{2} dx dy$$



$$1. \lim_{x \rightarrow 0} \frac{\sin 4x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{2x} = 2$$

$$2. \lim_{x \rightarrow 0} \frac{x \arcsin x}{x^3} = \lim_{t \rightarrow 0} \frac{\sin t}{\sin^3 t} = \lim_{t \rightarrow 0} \frac{\cos t}{3t^2} = \lim_{t \rightarrow 0} \frac{\frac{1}{2}t^2}{3t^2} = \frac{1}{6}$$

$$3. y = e^{x^2 \ln \frac{x}{2}} \cdot y' = e^{x^2 \ln \frac{x}{2}} \cdot 2x \ln \frac{x}{2} + x^2 \cdot \frac{1}{x} \cdot \frac{1}{x} = \frac{2x}{1} \ln \frac{x}{2} + \frac{1}{x}$$

$$4. x' = 2t \cos t - t \sin t = \cos t - t^2 \sin t \cdot y' = 2 \sin t$$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{1}{t} \cdot \frac{d^2 y}{dx^2} = \frac{d}{dt} \frac{dy}{dx} = \frac{1}{2t^2} \cdot 2 \sin t$$

$$1. \int \frac{x}{x\sqrt{x-2}} dx = \int \frac{1}{\sqrt{x-2}} dx = \frac{dx}{x\sqrt{x-2}} = \frac{2\sqrt{x-2}}{x-2} = \frac{dx}{2\sqrt{x-2}} = \sqrt{2} \arctan \frac{\sqrt{x-2}}{2} + C$$

$$2. \int_0^{\frac{1}{2}} x \arcsin x dx = \int_0^{\frac{\pi}{6}} u \cos u du = \int_0^{\frac{\pi}{6}} \frac{1}{2} u \sin 2u du$$

$$\frac{1}{4} u \cos 2u - \frac{1}{8} \sin 2u \Big|_0^{\frac{\pi}{6}} = \frac{3\sqrt{3}}{48} - \frac{\pi}{48}$$

$$0 \leq x < 1. \int_0^x f(t) dt = \int_0^x e^t dt = e^x - 1$$

$$x > 1. \int_0^x f(t) dt = \int_0^1 e^x dx + \int_1^x \frac{1}{x} dx = e - 1 + \ln x$$

$$e - 1 + \ln x, x > 1$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} (x + 1) = 2$$

$$V = \int_0^{2\pi} y^2 t \, dx = 8\pi \int_0^{2\pi} (1 - \cos t)^3 dt = 8\pi \int_0^{2\pi} (2\sin^2 \frac{t}{2})^3 dt = 2^6 \pi \int_0^{2\pi} \sin^6 \frac{t}{2} dt$$

$$\stackrel{2u=t}{=} 2^6 \pi \int_0^{\pi} \sin^6 u \, du = 2^7 \pi \int_0^{\frac{\pi}{2}} \sin^6 u \, du = \frac{5}{6} \frac{3}{4} \frac{1}{2} \frac{\pi}{2} = 2^7 \pi \cdot \frac{5}{16} = 20\pi^2$$

$$F(t) = \int_0^t f(t) \, dt$$

$$F(0) = \int_0^0 f(t) \, dt = 0, F(1) = \int_0^1 f(t) \, dt = 1$$

$$F'(t) = f(t) \Rightarrow F'(1) = f(1) = 0, 1$$

$$\int_a^b f(x) \, dx = f(\xi)(b-a), \quad \xi \in (a,b)$$

$$f'(\xi) = 0$$

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1. $\lim_{x \rightarrow 0} \sin x \sin \frac{1}{x} = 0$.

2. D

3. $p = 0, \int_0^1 \frac{1}{x} dx$

$p = 2, \int_0^1 \frac{dx}{x^{1-p}} = \int_0^1 x dx = \frac{1}{2} \quad D$

4.0

$$1. \lim_{x \rightarrow 0} \frac{\cos x - 1}{1 - x^{2/3} - 1} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}x^2}{\frac{1}{3}x^2} = \frac{3}{2}$$

$$2. y' = \frac{1}{x} - 2x = \frac{1}{2} - \frac{1}{2}, \ln 2 \quad y = 2x - 3$$

$$3. \int \frac{\sin x dy - y \cos x dx}{\sin x - y} = \int \frac{\sin x - y}{\sin x - y} dx = \int 1 dx = x + C$$

$$4. \int e^{\sin^2 x} \sin 2x dx = \int e^{\sin^2 x} d(1 - \sin^2 x) = e^{1 - \sin^2 x} C$$

$$5. \int_0^x f(t) dt = \int_0^3 1 - x^2 dx = \left[x - \frac{1}{3}x^3 \right]_0^3 = 6$$

$$6. V = \pi \int_a^b f(x) g(x)^2 dx$$

$$1. \lim_{x \rightarrow \infty} 1 - \frac{1}{x^2} = 1, \lim_{x \rightarrow \infty} e^{3x \ln 1 - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} e^{3x \frac{1}{x^2}} = e^0 = 1$$

$$2. x' = e^t \sin t - e^t \cos t, y' = e^t \sin t + \cos t \quad \frac{dy}{dx} = \frac{y'}{x'} = \frac{\sin t + \cos t}{\cos t - \sin t}$$

$$\frac{d^2 y}{dx^2} = \frac{d \frac{dy}{dx}}{dt \frac{dx}{dt}} = \frac{\sin t - \cos t^2 - \sin t + \cos t^2}{\cos t - \sin t^2} \cdot \frac{1}{e^t \cos t - \sin t} = \frac{2}{e^t \cos t - \sin t^3}$$

$$3. \lim_{x \rightarrow 1} \frac{x^x - t^{\frac{1}{t}} - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{x^x - e^{\frac{1}{x}} - 1}{x - 1} = \lim_{u \rightarrow 0} \frac{e^u - 1 - u}{u^2} = \lim_{u \rightarrow 0} \frac{e^u - 1}{2u} = \frac{1}{2}$$

$$1. \int_0^{\frac{\pi^2}{4}} \sin \sqrt{x} dx = \int_0^{\sqrt{x}} 2t \sin t dt = 2t \cos t \Big|_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} \cos t dt = \pi - 2$$

$$\ln f(x) = 3 \ln 1 - x + 2 \ln x - 1$$

$$\frac{f'(x)}{f(x)} = \frac{3}{1-x} - \frac{2}{x} - \frac{x-5}{x^2-1} \quad f'(x) = \frac{x-1^2}{x-1^2} \frac{x-5}{x-1} = 0 \quad 1-x-5$$

$$f'(x) = 0 \quad x-5 \quad x-1 \quad f(x) = f(5)$$

$$\ln f'(x) = 2 \ln 1 - x + \ln x - 5 = 3 \ln x - 1$$

$$\frac{f''(x)}{f'(x)} = \frac{2}{1-x} + \frac{1}{x^2} - \frac{3}{x-1} = \frac{24}{x-1} \quad f''(x) = \frac{24}{x-1} \frac{x-1^2}{x-1^3} = \frac{24x-1}{x-1^4} = 0 \quad x-1 \quad f(1)$$

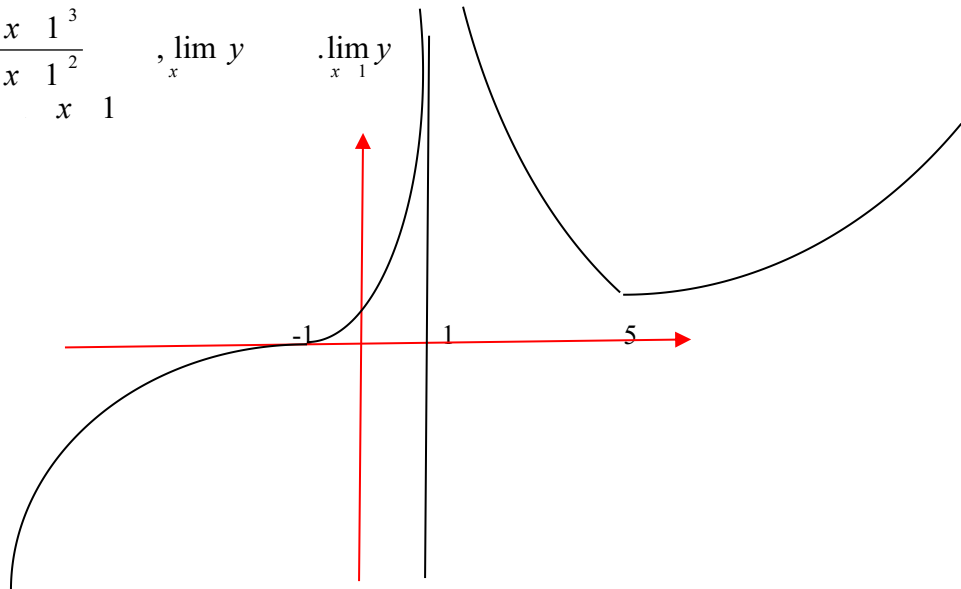
x	(- , -1)	-1	(-1,1)	1	(1,5)	5	(5,+)

$$k \lim_{x \rightarrow -1} \frac{y}{x} = \lim_{x \rightarrow -1} \frac{x^3 - 1}{x^2 - 1} \quad 1.b \lim_{x \rightarrow 1} y = kx = \lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 - 1}$$

$$\lim_{x \rightarrow 5} \frac{x^3 - 3x^2 + 1}{x^2 - 1} = \frac{125 - 45 + 1}{25 - 1} = \frac{81}{24} = \frac{27}{8}$$

$$y = x^5 \cdot f(1) = 0 \cdot f(5) = \frac{6^3}{4^2}$$

$$\lim_{x \rightarrow 1} y = \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}, \lim_{x \rightarrow 1} y = \lim_{x \rightarrow 1} y$$



$$x' = \frac{\sin t}{\sqrt{x'^2 + y'^2}} dt, y' = \frac{t \cos t}{\sqrt{x'^2 + y'^2}} dt, \int_0^{2\pi} \frac{\sin t}{\sqrt{x'^2 + y'^2}} dt = \int_0^{2\pi} \frac{t \cos t}{\sqrt{x'^2 + y'^2}} dt = 2\pi^2$$

$$2. \int \frac{2x-3}{x^2-2x+5} dx = \int \frac{2x-2+1}{x^2-2x+5} dx = \int \frac{d(x^2-2x+5)}{x^2-2x+5} + \int \frac{1}{x^2-2x+5} dx$$

$$\ln|x^2-2x+5| + \frac{1}{2} \arctan \frac{x-1}{2} + C$$

$$f(x) = c, d \quad M, m$$

$$m \cdot f(c) = M \quad \alpha m \quad \alpha f(c) \quad \alpha M$$

$$m \int_a^d f(x) dx = M \int_c^d f(x) dx - \beta m \int_c^d f(x) dx + \beta M \int_c^d f(x) dx$$

$$\int_c^d f(x) dx = \frac{\beta M - m}{\beta M - m} \int_c^d f(x) dx \quad \xi \in (c, d)$$

$$F(x) = \frac{1}{a} \int_0^a f(x) dx - \int_0^1 f(x) dx - \frac{1}{a} \int_0^a f(x) dx + \int_0^a f(x) dx - \int_a^1 f(x) dx$$

$$F(x) = f(\xi_1) - a f(\xi_1) + 1 - a f(\xi_2) - \xi_1 - 0, a, \xi_2 - a, 1$$

$$F(x) = f(\xi_1) - f(\xi_2) - a f(\xi_2) - f(\xi_1) - a - 1 - f(\xi_2) - f(\xi_1)$$

$$f(x) \quad \xi_1, \xi_2, f(\xi_2) - f(\xi_1) - 0 - a - 1 - 0$$

$$F(x) = 0 - \frac{1}{a} \int_0^a f(x) dx - \int_0^1 f(x) dx$$